

Student's Name _____

Learning Trajectory Record: Counting

Materials: 15 cubes or other small manipulatives; shopping cart sheet; cloth; 80 sticks or straws; 6 counters or chips

Age	Level Name	Questions	Response
1-2	Pre-Counter	For Pre-Counter through Reciter (10) [b] Say: "How high can you count? Start at 1 and show me." If the student stops, you can prompt to continue once per stop, for example, "What comes next? Can you go higher?"	
	Chanter		
2	Reciter		
3	Reciter (10)	For Corresponder and Counter (Small Numbers) Place five cubes in a line. [b] Say: "Count these blocks and tell me how many there are." If the student repeated the last counting word or otherwise showed he or she understood that 5 was the number of cubes (for example, "1, 2, 3, 4, 5 blocks!"), then he or she has achieved this level. Otherwise, ask: "How many blocks are there?" The student must say <i>five</i> without needing to recount the cubes to be at this level.	
	Corresponder		
4	Counter (Small Numbers)	Place about ten cubes near the shopping cart. [b] Say: "Please help me put four blocks on your shopping cart." If the student counts past 4, remind him or her that the goal is four and allow him or her to try again one time.	
	Producer (Small Numbers)		
4-5	Counter (10)	Show the student eight cubes laid in a line. [b] Say: "Show me how you can count the blocks, and tell me how many there are." Student is allowed to move cubes. Cover the cubes immediately. [b] Say: "Now, I am covering them all! How many blocks are under here?" Student is not allowed to recount.	



Student's Name _____

Learning Trajectory Record: Counting

Age	Level Name	Questions	Response
5-6	Counter and Producer (10+)	Lay fifteen cubes in an unordered, scrambled arrangement. [b]Say: "Show me how you can count the blocks and tell me how many there are." Student is allowed to move the cubes. Place the shopping cart in front of the student and about 15 cubes nearby. [b]Say: "Put exactly 10 blocks on your shopping cart."	
	Counter Backward from 10	[b]Say: "Help me count backward from 10 to 0: 10, 9..." If no response follows, say: "What comes next?" You can say <i>keep going</i> to encourage the student to keep counting once per stop.	
6-7	Counter from N (N+1, N-1)	Remove all objects. [b]Say: "Let's practice some counting. Please count to 10, starting at 4." if student starts at 1, interrupt by saying: "Please start at 4 and count to 10." [b]Say: "Can you help me, what number comes right after 5?" Student is at this level if they answer immediately, without counting from 1.	
	Skip Counter by 10s to 100	[b]Say: "Let's practice counting by tens. Please count to 100 by 10s." Show 8 bundles of 10 sticks or other objects or pictures grouped into 10. [b]Say: "There are 10 sticks in each bundle. How many sticks are there in all?" (80)	
	Counter to 100	[b] Say: "Start at 28 and keep counting up." if you wish, stop the student at about 52.	

Student's Name _____

Learning Trajectory Record: Counting

Age	Level Name	Questions	Response
6-7	Counter On Using Patterns	[b] Say: "How much is 3 more than 5?" (8)	
	Skip Counter	[b] Say: "Let's practice some counting by fives. Please count to 50 by fives." Repeat, counting by twos to 30 Show 8 bundles of sticks or other objects or pictures grouped into fives. [b] Say: "There are 5 sticks in each bundle. How many sticks are there in all?" (40)	
	Counter of Imagined Items	Hide six plastic chips under a cloth, and place five more near the cloth. [b] Say: "There are 5 chips here and 6 under the napkin, how many in all?" (11)	
	Counter On Keeping Track	Remove all objects. [b] Say: "How many is 6 more than 8?" (14)	
	Counter of Quantitative Units/Place Value	Show six bundles of 10 sticks and five single sticks in a disorganized arrangement (or other objects or pictures grouped into tens and ones) [b] Say: "These are bundles of sticks with 10 in each bundle and some other sticks. How many sticks are there in all?" (65)	
	Counter to 200	[b] Say: "What number comes after 159?" (160)	



Student's Name _____

Learning Trajectory Record: Counting

Age	Level Name	Questions	Response
7+	Number Conserver	Ask student to count two rows of objects (for example, twelve cubes in each) that are laid out across from each other and agree they have the same number. Spread out one row. Student is not allowed to recount. [b] Say: “Do these rows both still have the same number, or does one have more?” (same number)	
	Counter Forward and Back	[b] Say: “What is 14 less than 63?” (49) Student should keep track, counting down four and then a jump of ten or vice versa	
8	Counter to 1000	[b] Say: “Start at 487 and keep counting up.” If you wish, stop the student at about 500.	

Student's Name _____

Learning Trajectory Record: Numerals

Materials: paper and pencil; box; 6 objects; numeral cards 0-9; dot cards 0-9; 17 connecting cubes

Age	Level Name	Questions	Response
3	Quantity Representer	Ask student to put three objects into a box and close the box. [b] Say: "Can you write or draw on this paper so we remember how many are in the box?"	
4	Numeral Representer	Show numeral cards (1-5) and the dot cards (1-5). [b] Say: "Match the numbers with the dot cards."	
4-5	Functional Numeral User	Work with the student to put one object under a sheet of paper (or a container), two under another, and so forth up to six. [b] Say: "What could you write on each so we remember how many there are?" After the student is finished, say: "Which one has four? Which one has the most?"	
6	Teen/Ten + Recognizer	[b] Say: "Read me these numbers and tell me what each means." (If necessary, have student show you one or more using connecting cubes in tens and ones). 13 (one ten, three ones) 17 (one ten, seven ones) 12 (one ten, two ones)	
6-7	Decade Number Identifier	[b] Say: "Read me these numbers and tell me, if I wanted to make these numbers with single connecting cubes and towers of the cubes in 10s how many towers and how many single cubes would I need?" 30 50 20	
7	Digit Identifier	[b] Say: "Read me these numbers and tell me, if I wanted to make these numbers with single connecting cubes and towers of the cubes in 10s how many towers and how many single cubes would I need?" 59 8 52 100	



Student's Name _____

Learning Trajectory Record: Comparing and Ordering Numbers

Materials: cubes (9 of one color and 11 of another color); 10 small toys; dot cards (1-6)

Age	Level Name	Questions	Response
2	Object Corresponder	Show student a row of six cubes of one color (red) and a pile of about eight cubes of another color (blue). [b] Say: "Can you put out as many blue cubes as there are red cubes?"	
	Perceptual Comparer	Show the student two cubes of one color and four cubes of another color. [b] Say: "Which pile has more?"	
2-3	First-Second Ordinal Counter	Place four toys lining up at a toy house or other object. [b] Say: "They are waiting in line. Which one is first in line? Which one is second in line?"	
	Nonverbal Comparer of Similar Items	Show three cubes arranged in a row and three cubes arranged in a triangular form. [b] Say: "What do you think? Do these two groups have the same number?"	
4	Nonverbal Comparer of Dissimilar Items	Show three cubes arranged in an upside down triangular form and four cubes arranged in a square. [b] Say: "What do you think? Do these two groups have the same number?"	
	Matching Comparer	From Matching Comparer through to Counting Comparer (Same Size)	
	Knows-to-Count Comparer Counting Comparer (Same Size)	Show the student five cubes of one color and six cubes of another color. [b] Say: "Are there the same numbers of cubes in each group or does one group have more?"	

Student's Name _____

Learning Trajectory Record: Comparing and Ordering Numbers

Age	Level Name	Questions	Response
5	Counting Comparer (5)	Put out a pile of four cubes and five counters. [b] Say: "Are there more blocks or more counters or is there the same number?"	
	Ordinal Counter	Place ten small toys lining up at a toy house or other object. [b] Say: "They are waiting in line. Which one is third in line?" Repeat with other ordinal numbers, in random order, including fourth to tenth.	
6	Counting Comparer (10)	Show the student nine cubes of one color (blue) and eleven cubes of another color (red). [b] Say: "Are there more blue cubes or red cubes or is there the same number?"	
	Mental Number Line to 10	Have no objects available. [b] Say: "Which number is closer to 7: 3 or 4?" [b] Say: "What number is between 5 and 7?"	
	Serial Orderer to 6+	Show the student unordered dot cards from 1 to 6. [b] Say: "Please put these in order from smallest to largest."	
7	Place Value Comparer	[b] Say: "Which is more: 63 or 59? Explain how you know."	
	Mental Number Line to 100	[b] Say: "Which is closer to 45: 30 or 50?"	
8+	Mental Number Line to 1000s	[b] Say: "Which is closer to 3500: 2199 or 5004?"	



Student's Name _____

Learning Trajectory Record: Recognizing Number and Subitizing

Materials: 43 connecting cubes; cloth

Age	Level Name	Questions	Response
2	Small Collection Namer	Show the student two cubes. [b] Say: "How many blocks are there here?"	
3	Nonverbal Subitizer Maker of Small Collections	Place six cubes near the student. Put three cubes in front of you for two seconds, and then hide them. [b] Say: "Can you make the same number of blocks?"	
4	Perceptual Subitizer to 4	Show four cubes to the student for two seconds, and then remove them [b] Say: "How many?" (4)	
5	Perceptual Subitizer to 5	Show five cubes to the student for two seconds, and then remove them [b] Say: "How many?" (5)	
	Conceptual Subitizer to 10	Show ten cubes to the student, seven in one hand and three in the other, for two seconds, and then remove them [b] Say: "How many?" (10)	
6	Conceptual Subitizer to 20	Show three groups of five cubes for two seconds, and then remove them [b] Say: "How many?" (15)	
7	Conceptual Subitizer with Place Value and Skip Counting	Show four groups of ten and three single cubes for two seconds, and then remove them [b] Say: "How many?" (43)	
8+	Conceptual Subitizer with Place Value and Multiplication	Show three groups of ten and four groups of three single cubes for four seconds, and then remove them [b] Say: "How many?" (42)	

Student's Name _____

Learning Trajectory Record: Composing Number

Materials: paper and pencil; 10 cubes; cloth

Age	Level Name	Questions	Response												
4	Pre-Part Whole Recognizer	<p>Use the same task just with different numbers for all levels from Composer to 4, then 5 to Composer to 10. The task for composer to 5 is as follows: [b] Say: "I put five blocks here. Count with me. 1, 2, 3, 4, 5. 5!" Lay blocks in a straight line as you count. Say: "Now, I'm going to hide some." Cover the cubes with a cloth or hide them from the students' view with a sheet of paper; then hide three; then remove the cover to show the remaining two. [b] Say: "How many am I hiding?" If the student is correct repeat the process but hide 1. Students must answer all questions correctly and quickly to be at that level. Repeat with seven and ten blocks if student is successful with smaller numbers.</p>													
5	Inexact Part-Whole Recognizer														
	Composer to 4, then 5														
6	Composer to 7														
	Composer to 10														
7	+/- Fact Fluency to 20	<p>Student must write or say the answers to these addition and subtraction problems in less than one minute (or read each problem to the student, and have him or her answer within two seconds) to be at this level. [b] Say: "Tell me the answers to these problems as fast as you can."</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td>$7 + 8$</td> <td>$7 + 5$</td> <td>$16 - 8$</td> <td>$12 - 5$</td> </tr> <tr> <td>$17 - 9$</td> <td>$13 - 6$</td> <td>$2 + 9$</td> <td></td> </tr> <tr> <td>$5 + 9$</td> <td>$8 + 4$</td> <td>$3 + 8$</td> <td></td> </tr> </table>	$7 + 8$	$7 + 5$	$16 - 8$	$12 - 5$	$17 - 9$	$13 - 6$	$2 + 9$		$5 + 9$	$8 + 4$	$3 + 8$		
$7 + 8$	$7 + 5$	$16 - 8$	$12 - 5$												
$17 - 9$	$13 - 6$	$2 + 9$													
$5 + 9$	$8 + 4$	$3 + 8$													
	Composer with Tens and Ones	<p>[b] Say: "Show me how you can add 17 and 36 without paper and pencil." To be at this level, the student must compose numbers with both tens and ones. For example, <i>"17 and 36 is like 17 and 3, which is 20, and 33, which is 53."</i></p>													



Student's Name _____

Learning Trajectory Record: Adding and Subtracting

Materials: 54 connecting cubes; cloth; paper and pencil

Age	Level Name	Questions	Response
1	Pre +/–	Show two cubes, and then cover them with a cloth. Tell student to watch carefully, and then put one more cube under the cloth.	
3	Nonverbal +/–	Give the student six cubes [b] Say: “Make a group of blocks look just like I have.” (Gesture to the cloth.)	
4	Small Number +/–	[b] Say: “You have two balls and get one more. How many do you have in all?” (3)	
5	Find Result +/–	[b] Say: “You have two red balls and three blue balls. How many in all?” (5) [b] Say: “You have five balls and give two to Tom. How many do you have left?” (3)	
	Find Change +/–	[b] Say: “You have five balls and then get some more. Now you have seven in all. How many did you get?” (2) [b] Say: “There are six dogs and four balls. If we give a ball to each dog, how many dogs won’t get a ball?” (2)	
	Make It N +/–	[b] Say: “You have five apples but want to have seven. How do you make it seven?” (add 2)	
6	Counting Strategies +/–	Have no objects available. Allow, but do not mention or encourage, use of fingers. [b] Say: “How much is 6 + 7?” (13) [b] Say: “You have seven apples. How many more would you need to have 12?” (5)	

Student's Name _____

Learning Trajectory Record: Adding and Subtracting

Age	Level Name	Questions	Response
6	Part-Whole +/-	<p>[b] Say: "You have some balls, then you get 4 more balls, now you have 9. How many did you have to start with?" (5)</p> <p>Student is at Part-Whole +/- level if he or she used trial and error. Student is at Numbers-in-Numbers +/- level if he or she uses strategies such as: counts, putting up fingers, "Five, six, seven, eight, nine." Looks at fingers, and says, "Five!"</p>	
	Numbers-in-Numbers +/-		
7	Deriver +/-	<p>[b] Say: "What's 7 plus 8? Show how you figured this out." (15)</p> <p>Student must use derived fact to be at this level. For example, $7 + 8 = 7 + (7 + 1) = (7 + 7) + 1 = 14 + 1 = 15$; or $8 + 2 = 10$, so separate 7 into 2 and 5, add 2 and 8 to make 10, then add 5 more, 15.</p> <p>[b] Say: "What's 20 + 34?" (54)</p> <p>Student can use connecting cubes. For example, count up from 20; 30, 40, 50, plus 4 is 54.</p>	
8+	Problem Solver +/-	<p>[b] Say: "If I have 13 and you have 9, how many would I have to give you so that we had the same number?" (2)</p> <p>[b] Say: "What's 28 + 35?" (63)</p> <p>Student must operate strategically to be at this level.</p>	
	Multidigit +/-	<p>[b] Say: "What's 37 - 18?" (19)</p> <p>Student must use composition of tens and ones and other strategies to be at this level. For example, standard algorithm with explanation of why it works, or strategy such as "I take 1 ten off the 3 tens; that's 2 tens. I take 7 off the 7. That's 2 tens and zero...20. I have one more to take off. That's 19."</p> <p>[b] Say: "What's 28 + 35?" (63) Student may say, for example, "30 + 35 would be 65. But 28 is 2 less, so 63.</p>	
	Algorithm Computer +/-	<p>[b] Say: "Solve these problems. Show your work." A) $543 + 392 = ?$ (935) B) $797 - 429 = ?$ (305)</p>	

Student's Name _____

Learning Trajectory Record: Length Measurement

Materials: two pencils of different lengths; 12 inch or longer piece of string; 52 connecting cubes; two books of different sizes; ruler

Age	Level Name	Questions	Response
3	Length Quantity Recognizer	Give the student two pencils of different lengths. [b] Say: "Which of these pencils is longer?"	
4	Length Direct Comparer		
5	Indirect Length Comparer	Give the student two pencils of different lengths, laid about a foot apart, and a piece of string. [b] Say: "Which of these pencils is longer? Tell me without moving the pencils. You can use this string to help you."	
	Serial Orderer to 6+	Place connecting cube towers from 3 to 10 in random order. Student is not allowed to take them apart or connect them. Say: "Put these in order from smallest to largest. Line them up so their bottoms are at the edge of the table."	
6	End-to-End Length Measurer	Give the student a book and a pile of cubes. [b] Say: "Would you please measure how long this book is with these cubes?"	
7	Length Unit Relater and Repeater	Give the student a different size book and a ruler. [b] Say: "Would you please measure how long this book is with this ruler?"	
8	Length Measurer	Show the student the piece of string (12 inches or longer). [b] Say: "How long is this string, measured in inches?" Student needs to measure accurately, measuring from the zero point on the ruler and considering fractional parts of a unit to be at this level. Show the student a "zig-zag" path consisting of straight line segments and repeat as above.	
	Conceptual Ruler Measurer	[b] Say: "How many meters long do you estimate this wall is?" Student must show that they can visualize a meter and mentally repeat its length along the wall fairly accurately.	

Student's Name _____

Learning Trajectory Record: Recognizing Geometric Shapes

Materials: shape sets in two colors (yellow and blue); straws

Age	Level Name	Questions	Response
2	Shape Matcher —Identical —Sizes —Orientations	—Identical Display shape C from the blue Shape Set. Under it display shapes B, T, C, and N from the yellow Shape Set. [b] Say: “Which yellow shape (gesture) matches this blue shape (gesture)?” (C) —Sizes Display shape G from the blue Shape Set. Under it display shapes E, I, Z7, and Z1 from the yellow Shape Set. [b] Say: “Which yellow shape (gesture) is the same shape as this blue shape (gesture)?” (I) —Orientations Display shape C from the blue Shape Set, turn 45°. Under it display shapes W, C, A, and B from the yellow Shape Set. [b] Say: “Which yellow shape (gesture) matches this blue shape (gesture)?” (C)	
3	Shape Recognizer—Typical	Show the following shapes from a Shape Set to the student. [b] Say: “What is this called?” Circle: C Square: S Triangle: D	
	Shape Matcher—More Shapes —Sizes and Orientations	Display shape M from the blue Shape Set, with the long side horizontal at the top, set as upside down triangle. Under it, display shapes B, I (set as right side up triangle), W, and Z1 from the yellow Shape Set. [b] Say: “Which yellow shape (gesture) is the same shape as this blue shape (gesture)?” (I)	



Student's Name _____

Learning Trajectory Record: Recognizing Geometric Shapes

Age	Level Name	Questions	Response
4	Shape Recognizer—Circles, Squares, and Triangles	Show the following shapes from a Shape Set to the student. [b] Say: “What is this called?” Square: C, turned (set as diamond) Triangles: W, X, Y	
	Constructor of Shapes from Parts—Looks Like	[b] Say: “We’re going to use these straws to make shapes. Can you make a triangle using some of the straws?” Check whether the student can at least approximate a triangle; for example create a triangle with side lines extending past base.	
5	Shape Recognizer—All Rectangles	Show the following shapes from a Shape Set to the student [b] Say: “What is this called?” Rectangles: T, U, V, Z1	
	Side Recognizer	Show shape G from Shape Set to the student. [b] Say: “Show me the sides of this shape.”	
	Angle (Corner) Recognizer	Show shape G from Shape Set to the student. [b] Say: “Show me the angles of this shape.”	
	Shape Recognizer—More Shapes	Display all the Shape Sets shapes of one color in random order. Name the following shape categories one at a time. [b] Say: “Can you find all the [wolf]?” Shape categories: Rectangles, Triangles, Trapezoids, Rhombuses, Hexagons	
6	Shape Identifier	Show the following shapes from a Shape Set to the student. Say: “What is this called?” Shapes: Rectangles, Triangles, Trapezoids, Rhombuses, Hexagons	

Student's Name _____

Learning Trajectory Record: Recognizing Geometric Shapes

Age	Level Name	Questions	Response
7	Parts of Shapes Identifier	<p>Show student a obtuse (very "skinny") triangle from a Shape Set or draw one.</p> <p>[b] Say: "To what category of shapes would this shape belong? Why?" (triangles)</p> <p>Student needs to justify his or her response to be at this level; for example, "No matter how skinny it looks, that's a triangle because it has 3 sides and 3 angles."</p>	
	Constructor of Shapes from Parts—Exact	<p>[b] Say: "We're going to use these straws to make shapes. Can you make a rectangle using some of the straws?"</p> <p>Check the accuracy of the rectangle. Student must make a shape that is completely correct, based on knowledge of components and relationships.</p>	
8	Shape Class Identifier	<p>Give student one set of Shape Set shapes.</p> <p>[b] Say: "Can you sort these shapes into a small number of categories?"</p> <p>Student must use geometric categories; for example, "I put the triangles over here, and the quadrilaterals, including squares, rectangles, rhombuses, and trapezoids over there."</p>	
	Shape Property Identifier	<p>Give student one set of Shape Set shapes.</p> <p>[b] Say: "Can you sort these shapes into categories based on the properties of the shapes?"</p> <p>Student must use geometric categories; for example, "I put the shapes with opposite sides parallel over here, and those with four sides but not both pairs of sides parallel over there."</p>	



Student's Name _____

Learning Trajectory Record: Composing Geometric Shapes

Materials: pattern block puzzles and pattern blocks

1. Give the student the puzzle and the Pattern Blocks, randomly mixed in front of them.
2. Say: **“Use Pattern Blocks to fill this puzzle. Put them together with full sides touching.”**

Age	Level Name	Questions	Response
2	Pre-Composer	For Pre-Composer through Piece Assembler use puzzle and the pattern blocks.	
4	Piece Assembler	At the Piece Assembler level, the student is able to match most of the shapes, but may have 1–2 gaps and “hangovers.”	
5	Picture Maker	To succeed at this level, student must completely and accurately cover the puzzle (with no gaps or “hangovers”). Students may use trial and error to do so.	
	Shape Composer	Student solves the puzzle systematically, immediately, and confidently. They appear to search for “just the right shape” that they “know will fit” and then find and place it.	
6	Substitution Composer	As with the previous, students solve the puzzle systematically, immediately, and confidently. They appear to search for “just the right shape” that they “know will fit” and then find and place it. For this level, check whether students fill the hexagons in different ways. If not, have them complete the puzzle, then challenge them to do it a different way. (You may have to put the yellow hexagons away.)	

Student's Name _____

Learning Trajectory Record: Comparing Geometric Shapes

Materials: shape sets in two colors (blue and yellow)

Age	Level Name	Questions	Response
3	"Same Thing" Comparer	<p>For Part Comparer through to Congruence Determiner. The early levels are not instructional goals, they are developmental levels that many children display. Congruence Determiner is the first level that involves the mathematical thinking children develop.</p> <p>[b] Say: "Find some yellow shapes that match the blue shapes exactly."</p> <p>Once the student has attempted 3 or 4 matches, have the student stop. How the student compared the shapes indicates their level:</p> <p>Part Comparer — May say two shapes are the same after matching one side on each.</p> <p>Some Attributes Comparer — Looks for differences in attributes, but may examine only part of shape. (e.g., "These are the same," indicating the top halves of the shapes are similar by laying them on top of each other.)</p> <p>Most Attributes Comparer — Looks for differences in attributes, examining full shapes, but may ignore some spatial relationships.</p> <p>Congruence Determiner — Determines congruence by comparing all attributes and all spatial relationships. For example, says that two shapes are the same shape and the same size after comparing every one of their sides and angles.</p> <p>Congruence Superposer Comparing — Moves and places objects on top of each other to determine congruence.</p> <p>Congruence Representer — Refers to geometric properties and explains with transformations. For example, says these must be congruent, because they have equal sides, all square corners, and I can move them on top of each other exactly.</p>	
4	"Similar" Comparer		
4	Part Comparer		
	Some Attributes Comparer		
5	Most Attributes Comparer		
7	Congruence Determiner		
8	Congruence Representer		



Student's Name _____

Learning Trajectory Record: Spatial Sense and Motions

Materials: shape sets or pattern blocks and geometric or pattern block puzzles (other shape manipulatives are optional)

Age	Level Name	Questions	Response
4	Simple Turner	Give the student Shape Sets to explore, and observe how he or she manipulates them. Show two of the same shape. [b]Say: "Are these the same? Show me."	
5	Beginning Slider, Flipper, Turner		
6	Slider, Flipper, Turner	As the student completes pattern block puzzles, hopefully increasing in difficulty, work on a puzzle of your own and pretend to struggle with a few shapes. [b]Say: "Can you help me complete this puzzle? How?" Tell the student to finish the puzzle, and observe his or her work. [b]Say: "What do you think of these shapes? Tell me how you got them to fit. How did you know how to move them?"	
7	Diagonal Mover		
8	Mental Mover		

Student's Name _____

Learning Trajectory Record: Patterns & Pre-Algebraic Thinking

Materials: pattern blocks; connecting cubes (5 red, 5 white, 5 blue); paper and pencil

Age	Level Name	Questions	Response
2	Pre-Explicit Patternner	Informal observation is a good strategy to see whether students can recognize patterns. You can also show a pattern and ask what they see. Lay out the ABABAB pattern of pattern blocks (red square, green triangle...six blocks in all) with one missing (ABAB_BAB). In front of the student, lay out six red squares and six green triangles in random arrangement. [b]Say: "I made a pattern with these blocks but one block is missing. Can you fix this pattern?"	
3	Pattern Recognizer		
3-4	Pattern Fixer		



Student's Name _____

Learning Trajectory Record: Patterns & Pre-Algebraic Thinking

Age	Level Name	Questions	Response
4	Pattern Duplicator AB	Lay out the ABABAB pattern of pattern blocks (red square, green triangle...six blocks in all). In front of the student, place six red squares and six green triangles in random arrangement. [b]Say: "I made a pattern with these blocks. Please make the same kind of pattern here using these blocks." Gesture in front of the student.	
	Pattern Extender AB	Lay out the ABABAB pattern of pattern blocks (red square, green triangle...six blocks in all). In front of the student, place six red squares and six green triangles in random arrangement. [b]Say: "I made a pattern with these blocks. Can you finish my pattern here the way I would?" Gesture to the right of the pattern.	
	Pattern Duplicator	Lay out the ABBABB pattern of pattern blocks (red square, green triangle, green triangle...nine blocks in all). In front of the student, place six red squares and six green triangles in random arrangement. [b]Say: "I made a pattern with these blocks. Please make the same kind of pattern here, using these blocks." Gesture in front of the student.	
5	Pattern Extender	Lay out the ABBABB pattern of pattern blocks (red square, green triangle, green triangle...nine blocks in all). In front of the student, place six red squares and six green triangles in random arrangement. [b]Say: "I made a pattern with these blocks. Can you finish my pattern here the way I would?" Gesture to the right of the pattern.	

Student's Name _____

Learning Trajectory Record: Patterns & Pre-Algebraic Thinking

Age	Level Name	Questions	Response
6	Pattern Unit Recognizer	<p>Show a tower in an ABCABC pattern (red, white, blue...six cubes in all). Place <i>unconnected</i> cubes, three red, three white, and three blue, in front of the student.</p> <p>[b] Say: "I want to make my tower bigger and keep the same pattern but I don't have any more small towers. Can you make a small tower with these pieces and keep the same pattern?"</p>	
7	Numeric Patterner	<p>Make an ABCABCABC pattern (for example, with pattern blocks: square, triangle, hexagon, square, triangle, hexagon...) and label the shapes 1, 2, 3, 4, 5, 6, 7, 8, 9.</p> <p>Say: "If we continue this pattern, what will shape number 30 be?" (C/hexagon)</p>	
	Beginning Arithmetic Patterner	<p>Show a number sentence such as $23 + 7 - 7 = [wo]$</p> <p>[b] Say: "What number can you put in the box to make this number sentence true? How do you know?" (23)</p>	
8	Relational Thinker +/-	<p>Show a number sentence such as $6 + 8 = [wo] + 9$</p> <p>[b] Say: "What number can you put in the box to make this number sentence true? How do you know?" (5)</p> <p>Repeat with number sentences such as $90 + 62 + 19 = [wo]$ (171)</p>	
	Relational Thinker — Symbolic +/-	<p>Show $a + b = b + a$</p> <p>[b] Say: "Will this number sentence always be true no matter what numbers you use for a and b? How do you know?" (yes)</p> <p>Show $a - b = b - a$</p> <p>[b] Say: "Will this number sentence always be true no matter what numbers you use for a and b? How do you know?" (no)</p>	



Student's Name _____

Learning Trajectory Record: Patterns & Pre-Algebraic Thinking

Age	Level Name	Questions	Response
9	Relational Thinker with Multiplication	Show $7 \times 6 + 6 = 8 \times 6$ [b] Say: "Can you tell if this is true without doing all the arithmetic? How do you know?" (yes)	
	Relational Thinker	Show $8 \times (7 + 3) = [wol]$ [b] Say: "Can you tell me what goes into the box to make this sentence true?" (80)	

Student's Name _____

Date _____

Trajectory Progress Chart: Number

Age Range	Counting	Numerals	Comparing and Ordering Number	Recognizing Number and Subitizing (Instantly Recognizing)	Composing Number (Knowing Combinations of Numbers)	Adding and Subtracting (Counting-Based Strategies)
1 year	Pre-Counter Chanter					Pre +/-
2	Reciter		Object Corresponder Perceptual Comparer First-Second Ordinal Counter	Small Collection Namer		
3	Reciter (10) Corresponder	Quantity Representer	Nonverbal Comparer of Similar Items (1-4 items) Nonverbal Comparer of Dissimilar Items	Nonverbal Subitizer Maker of Small Collections		Nonverbal +/-
4	Counter (Small Numbers) Producer (Small Numbers) Counter (10)	Numeral Representer Functional Numeral User	Matching Comparer Knows-to-Count Comparer Counting Comparer (Same Size)	Perceptual Subitizer to 4	Pre-Part-Whole Recognizer	Small Number +/-
5	Counter and Producer (10+) Counter Backward from 10		Counting Comparer (5) Ordinal Counter	Perceptual Subitizer to 5 Conceptual Subitizer to 10	Inexact Part Whole Recognizer Composer to 4, then 5	Find Result +/- Find Change +/- Make it N +/-
6	Counter from N (N + 1, N - 1) Skip Counter by 10s to 100 Counter to 100 Counter On Using Patterns Skip Counter Counter of Imagined Items Counter On Keeping Track Counter of Quantitative Units Counter to 200	Teen/Ten + Recognizer Decade Number Identifier	Counting Comparer (10) Mental Number Line to 10 Serial Orderer to 6+	Conceptual Subitizer to 20	Composer to 7 Composer to 10	Counting Strategies +/- Part-Whole +/- Numbers-in-Numbers +/-



Age Range	Counting	Numerals	Comparing and Ordering Number	Recognizing Number and Subitizing (Instantly Recognizing)	Composing Number (Knowing Combinations of Numbers)	Adding and Subtracting (Counting-Based Strategies)
7	Number Conserver Counter Forward and Back		Place Value Comparer Mental Number Line to 100	Conceptual Subitizer with Place Value and Skip Counting	+/- Fact Fluency to 20 Composer with Tens and Ones	Deriver +/-
8+			Mental Number Line to 1000s	Conceptual Subitizer with Place Value and Multiplication		Problem Solver +/- Multidigit +/- Algorithm Computer

Student's Name _____

Date _____

Trajectory Progress Chart: Geometry

Age Range	Length Measurement	Recognizing Geometric Shapes	Composing Geometric Shapes	Comparing Geometric Shapes	Spatial Sense and Motions	Patterning & Pre-Algebraic Thinking
2 years		Shape Matcher —Identical —Sizes —Orientations				Pre-Explicit Patterner
3	Length Quantity Recognizer	Shape Recognizer —Typical Shape Matcher —More Shapes —Sizes and Orientations	Pre-Composer	“Same Thing” Comparer		Pattern Recognizer
4	Length Direct Comparer	Shape Recognizer —Circles, Squares, and Triangles Constructor of Shapes from Parts—Looks Like	Piece Assembler	“Similar” Comparer Part Comparer Some Attributes Comparer	Simple Turner	Pattern Fixer Pattern Duplicator AB Pattern Extender AB Pattern Duplicator
5	Indirect Length Comparer	Shape Recognizer —All Rectangles Side Recognizer Angle (Corner) Recognizer Shape Recognizer —More Shapes	Picture Maker Shape Composer	Most Attributes Comparer	Beginning Slider, Flipper, Turner	Pattern Extender
6	End-to-End Length Measurer	Shape Identifier	Substitution Composer		Slider, Flipper, Turner	Pattern Unit Recognizer
7	Length Unit Relater and Repeater	Parts of Shapes Identifier Constructor of Shapes from Parts—Exact		Congruence Determiner	Diagonal Mover	Numeric Patterner Beginning Arithmetic Patterner

Age Range	Length Measurement	Recognizing Geometric Shapes	Composing Geometric Shapes	Comparing Geometric Shapes	Spatial Sense and Motions	Patterning & Pre-Algebraic Thinking
8+	Length Measurer Conceptual Ruler Measurer	Shape Class Identifier Shape Property Identifier		Congruence Representer	Mental Mover	Relational Thinker — Symbolic +/— Relational Thinker +/—
9						Relational Thinker with Multiplication Relational Thinker